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Generalized Variance Chart for Multivariate Quality Control Process Procedure with Application

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Abstract

Generalized variance|S|quality control chart is very powerful way to detect small shifts in the mean vector. The main purpose of this paper, presents an improved the generalized variance |S|quality control chart for multivariate process. Generalized variance chart allow us to simultaneously monitor whether joint variability of two or more related variables is in control. In addition, a control chart commonly requires samples with fixed size be taken at fixed intervals. It is assumed that in both univariate and multivariate control charts, each sample is independent of the previous samples. Industry fertilizers is important one of the chemical industries in Egypt, so that this work concerns the fertilizers industries quality control, especially urea fertilizer with application on Delta fertilizer and chemical industries which is considered one of the leading companies the field of fertilizer production in Middle east with application of multivariate quality control procedures to achieve best one procedure for multivariate quality control. This application shows that the company should use the multivariate quality control chart to determine whether not the process is in-control because the production have several correlated variables, and the used of separate control charts is misleading because the variables jointly affect the process. The used of separate univariate control charts in multivariate situation lead to a type I error and the probability of a point correctly plotting in-controlare not equal to their expected values.

Keywords: Quality Control, Multivariate Process, Generalized Variance

1. Introduction

The rapid growth data acquisition technology and the uses of online computers for process monitoring led to an increased interest in the simultaneous control of several related quality characteristics. These techniques are often referred to multivariate statistical process control procedures. The use of separate univariate control chart for each quality characteristic has proved to be inappropriate. This is because, it neglects the correlation between the multiple quality characteristics; and this leads to incorrect results.

Thus, this paper decides that there is a strong need for an applied work on the practical development and application study of generalized variance $|S|$ quality control chart for multivariate process.

The modern statistical process control took place when Walter A. Shewhart in 1926 developed the concept of a control chart based on the monitoring of the process mean level through sample mean ($\bar{X}$ chart) and process dispersion through sample range (R chart) or sample standard deviation chart. In the multivariate setting, Harold Hotelling (1931,1947) published what can be called the first major works in multivariate quality control. Hotelling developed the T^2 statistic and the statistics based on the sample variance-covariance matrix$S$procedure, and its extensions to control charts to combine measurements taken on variables in several dimensions into a single measure of excellence.

After Hotelling there was no significant work done in this field until the early sixties, when with the advances in computers, interest in multivariate statistical quality control was revived. Since then, some authors have done some work in this area of multivariate quality control.

The two more important and used statistics based on the sample variance-covariance matrix$S$, are Likelihood-Ratio and the generalized variance $|S|$, the determinant of S, Wilks, S. (1932); Anderson, T. W., (1958); Korin, B. P. (1968); Alt, F. A. (1985, 1998); Aparisi, F. et al (1999); Dogu, E. and Kocakoc, I. D.(2011). Alt (1985) proposed several control charts based on the sample generalized variance, denoted by$|S|$. One of these methods using the $|S|$ control chart is to utilize its distributional properties. The other method is constructed using the first two moments of the$|S|$and the property that most of the probability distribution of $|S|$ is contained in the interval:$E(|S|) \pm 3[V(|S|)]^{\frac{1}{2}}$.

Houshmand, A. and Javaheri, A. (1998) presented two procedures to control the covariance matrix in a multivariate setting. The advantages of these procedures are that they allow the investigators to identify the sources of the out-of-control signal. These procedures are based on constructing tolerance regions to control the parameters of the correlation matrix. Linna, Woodall (2001) presented a model for correlated quality variables with measurement error. The model determined the

performance of the multivariate control charting methods. The usual comparison of control chart performance do not directly Apply in the presence of measurement error. In this paper there is no consideration to the case where the covariance matrixes of either the quality variables or measurement errors are allowed to shift. Such charges obviously add another dimension of complexity to the issue of control chart performance. Also in this paper the context of known in-control mean vector and known covariance matrix were discussed.
Djauhari, M.A. (2005) proposed an improved control chart having unbiased control limits. This method focuses on use of determinate of the average of sample as an estimate of the true generalized variance and its square root. In this paper which refers to the use of $|S|$ and $\sqrt{|S|}$ as multivariate dispersion measures have some limitations. In general some changes of the covariance structure do not change the sample generalized variance and are not apt to be detected using$|S|$ and$\sqrt{|S|}$ chart. He presented these charts and improved their effectiveness by correcting the base of the control limits. He also discuss the problems encountered when he use the standard chants. He derived the bias of their control limits and suggests improvements.

2. Multivariate Quality Control Chart

Multivariate quality control charts are a type of variables control that how correlated, or dependent, variables jointly affect a process or outcome.The multivariate quality control charts are powerful and simple visual tools for determining whether the multivariate process is in-control or out-of-control. In other words, control charts can help us to determine whether the process average (center) and process variability (spread) are operating at constant levels. Control charts help us focus problem – solving efforts by distinguishing between common and assignable cause variation. Multivariate control chart plot statistical from more than one related measurement variable. The multivariate control chart shows how several variables jointly influence a process or outcome.
It is demonstrated that if the data include correlated variables the use of separate control chart is misleading because the variables jointly affect the process. If we use separate univariate control chart in a multivariate situation, type I error and probability of a point correctly plotting in- control are not equal to their expected values the distortion of those values increases with the number of measurement variables.

It is shown that multivariate control chart has several advantages in comparison with multiply univariate charts:

- The actual control region of the related variables is represented.
- We can maintain specification type I error.
- A signal control limit determines whether the process is in control.

- Multivariate control chart simultaneously monitor two or more correlated variables. To monitor more than one variable using univariate charts, we need to create a univariate charts for each variable.
- The scale on multivariate control charts unrelated to the scale of any of the variables.
- Out-of-control signals in multivariate charts do not reveal which variable or combination of variables cause the signal.

A multivariate control chart consists of:

- Plotted points, each for which represents a rational subgroup of data sampled from the process, such as a subgroup mean vector individual observation, or weighted statistic.
- A center line, which represents the expected value of the quality characteristics for all subgroups.
- Upper and lower control limits (*UCL* and *LCL*), which are set a distance above and below the center line. These control limits provide a visual display for the expected amount for variation. The control limits are based on the actual behavior of the process, not the desired behavior or specification limits. A process can be in control and yet not be capable of meeting requirements.

3. Multivariate Process Variability Control Chart

Monitoring process variability is an important part of any control procedure. Montgomery (2001, p. 532) point out that "just as it is monitor process variability". A similar remark is made by Alt and Simith (1998, p. 341). Events that need to take place for a successful important of statistical process control include an initial examination of stability and a capability analysis. Once this stage is satisfactorily completed, ongoing monitoring of key process parameters is necessary. One vital parameter is process variability, and the need to monitor and control variability motivates this paper.

It focuses on use of the determinate of the sample variance-covariance matrix $|S|$, also called the sample generalized variance.

Jackson (1988) proposed that the starting point of the statistical application of the method of principal components is the sample covariance matrix S. for a p-variate problem,

$$S = \begin{bmatrix} s_1^2 & s_{12} & \cdots & s_{1p} \\ s_{21} & s_2^2 & \cdots & s_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ s_{p1} & s_{p2} & \cdots & s_p^2 \end{bmatrix} \quad (1)$$

wheres_i^2 is the variance of the i^{th} variable and s_{ij} is the covariance between the i^{th} and j^{th} variables.

Process variability is summarized by the$p \times p$ covariance matrix. The main diagonal elements of this matrix are the variance of the individual process variability, and the off-diagonal elements are the covariances. If the covariances are not equal to zero, it indicates a relationship existing between those two variables, the strength of that relationship (if it is linear) being represented by the correlation:

$$r_{ij} = \frac{s_{ij}}{(s_i s_j)}$$

Alt and Smith (1998) presented two useful procedures variability, the first procedure is a direct extension of the univariateS^2 control chart, where S^2 is the sample variance.

The procedure is equivalent to repeated tests of significance of the hypothesis that the process covariance matrix is equal to particular matrix of constants $\sum$ .if this approach is used, the statistic plotted on the control chart for the i^{th} sample is

$$W_i = -pn + pn\ln(n) - n\ln\left(\frac{A_i}{|\Sigma|}\right) + tr\left(\Sigma^{-1}A_i\right), \tag{2}$$

where$A_i = (n-1)S_i$, $S_i$is the sample covariance matrix for sample *i*, and *tr* is the trace operator (the trace of matrix is the sum of the main diagonal elements) if the value of W_i plots above the upper control limit$UCL = \chi^2_{\alpha,\frac{p(p+1)}{2}}$, the process is out of control.

The second approach is based on the sample generalized variances, $|S|$. this statistic, which is the determinant of the sample covariance matrix, is a widely used measure of multivariate dispersion.

The$|S|$ chart, as presented by Montgomery (2001, pp. 533-534) have dependent on the mean variance of$|S|$- that is, $E(|S|)$ and $V(|S|)$ - and the property that most of the probability distribution of $|S|$ is contained in the interval: $E(|S|) \pm 3\sqrt{V(|S|)}$ as follows as:

$$\text{and}\left.\begin{array}{l} E(|S|) = b_1|\Sigma| \\ V(|S|) = b_2|\Sigma|^2 \end{array}\right\}, \tag{3}$$

where

$$b_1 = \frac{1}{(n-1)^p}\prod_{i=1}^{p}(n-i)$$

and

$$b_2 = \frac{1}{(n-1)^{2p}}\prod_{i=1}^{p}(n-i)\left[\prod_{j=1}^{p}(n-j+2) - \prod_{j=1}^{p}(n-j)\right]$$

Therefore, the parameters of the control chart for $|S|$ would be

$$\left.\begin{aligned} UCL &= |\Sigma|(b_1 + 3\sqrt{b_2}) \\ CL &= b_1|\Sigma| \\ LCL &= |\Sigma|(b_1 - 3\sqrt{b_2}) \end{aligned}\right\} \tag{4}$$

The lower control limit in equation (4) con replaced with zero if the calculated value is less than zero. He noted that, usually in practice Σ could be estimate by a sample covariance matrix S, based on the analysis of preliminary samples.

Accordingly, we should replace $|\Sigma|$ in equation (4) by $\frac{|\Sigma|}{b_1}$. Since equation (3) has shown that $\frac{|S|}{b_1}$ is an unbiased estimator of $|\Sigma|$.

As a result we find that the control chart parameters are

$$\left.\begin{aligned} UCL &= |S|(1 - \frac{3\sqrt{b_2}}{b_1}) \\ CL &= |S| \\ LCL &= |S|(1 - \frac{3\sqrt{b_2}}{b_1}) \end{aligned}\right\} \tag{5}$$

4. The Application

Delta Fertilizers and Chemical Industries are considered one of the leading companies in the field of fertilizers production in Egypt. About 4500 employees are working for it, on the various managerial levels. Urea production is one of the major products of the company. The production of urea occurs through three stages, summarized as follows:

4.1High Pressure Stage:

In this stage, urea is produced through two reactions; the first reaction occurs by condensation of Ammonia Gas and Carbon dioxide under high pressure and temperature for the sake of the production of intermediate material, known as Carbamate .The second reaction happens by separating the water from the Carbamate in order to a chive urea. In this, stage the condensation of urea approximately 56%.

It contains 16 variables, these are:

X_1	E-201Outlet Temperature
X_2	Outlet Cold NH3 from E- 201
X_3	Co2 to Train
X_4	Co2 pressure to Synthesis
X_5	Co2 after E-22
X_6	R-201
X_7	Temperature in Reactor R-201
X_8	Temperature in Reactor R-201
X_9	Temperature in Reactor R-201
X_{10}	Temperature in Reactor R-201
X_{11}	Stripper level
X_{12}	Liquid Leaving the Stripper
X_{13}	Stream from E-204 to j-201
X_{14}	Conditioned water to Scrubber E-204
X_{15}	Conditioned water from Scrubber E-204
X_{16}	Stream from j-203

Table analysis of laboratory in this stage:

$t_{1.1}$	NH_3	Rector outlet
$t_{1.2}$	CO_2	Rector outlet
$t_{1.3}$	UR	Rector outlet
$t_{1.4}$	B_1	Rector outlet
$t_{1.5}$	H_2O	Rector outlet
$t_{2.1}$	NH_3	Stripper outlet
$t_{2.2}$	CO_2	Stripper outlet
$t_{2.3}$	UR	Stripper outlet
$t_{2.4}$	B_1	Stripper outlet
$t_{2.5}$	H_2O	Stripper outlet

4.2 Low Pressure Stage:

In this stage, the condensation of urea liquid rises from 56% to 71%. This happens through the decomposition of the remaining Carbamateand the elimination of water under low pressure.

It contains seven variables, these are:

y_1	Urea Solution from stripper E-202
y_2	Steam to E-205
y_3	Urea Carbonate Solution from stripper T-201 to E-205
y_4	Gas leaving T-201
y_5	Level in TK-201
y_6	P-203
y_7	Urea Solution in TK-201

Table analysis of laboratory in this stage:

$t_{3.1}$	NH_3	D 202 Outlet
$t_{3.2}$	CO_2	D 202 Outlet
$t_{3\cdot 3}$	UR	D 202 Outlet
$t_{3.4}$	B_1	D 202 Outlet
$t_{3.5}$	H_2O	D 202 Outlet
$t_{4.1}$	NH_3	In TK 201
$t_{4.2}$	CO_2	In TK 201
$t_{4.3}$	UR	In TK 201
$t_{4.4}$	B_1	In TK 201
$t_{4.5}$	H_2O	In TK 201
$t_{5.1}$	NH_3	In PI 302
$t_{5.2}$	CO_2	In PI 302
$t_{5.3}$	UR	In PI 302

4.3 Evaporation and Prilling Stage:

This stage occurs by two stage:
Firstly, Evaporation stage:

In this stage, the condensation of urea rises from 71% to 98.7% approximately and the urea liquid trams forms to urea melt. This happens under high pressure and temperature.
Secondly, Prilling Stage:

In this stage, the urea melt is through formed into prilling in the prilling tower.

It contains four variables, these are:

Z_1	Urea Solution from D-204 to E-209
Z_2	D- 205 Vacuum
Z_3	Urea to periling tower X-202
Z_4	E- 211 Vacuum

Table analysis of laboratory in this stage:

$t_{6.1}$	B_1
$t_{6.2}$	H_2O
$t_{6.3}$	Pills > 3.35
$t_{6.4}$	Pills 3.35: 2.4
$t_{6.5}$	Pills 2.4 : 1.4
$t_{6.6}$	Pills 1.4 :1.0
$t_{6.7}$	Pills < 1.0
$t_{6.8}$	UR

4.4 Data Description:

For the application of multivariate quality control, chart data originate from urea production process, which consists of the three stages and the analysis of laboratory, which discussed above.

The number of the sample is 732 observations taken per hour.

The advantages of this sample that, it has several variables and several stage of the production. This advantage of the production is the basic reason for choosing this production to allow us to study the multivariate quality control charts.

In this application, we shall introduce the most common using technique of multivariate quality control chart; generalized variance chart.

A generalized variance chart consists of:

- Plotted points, each of which represents the generalized variance for each observation.
- A center line (green), which is the median of the theoretical distribution of generalized variance statistic.
- Control limits (red), which provide a visual means for assessing whether the process is in-control. The control limits represent the expected variation.

MINITAB marks points outside of the control limits with a red symbol.

(a)Generalized Variance Chart of X_1; ...; X_{16} and $t_{1.1}$;…;$t_{2.5}$

Test Results for Generalized Variance Chart of X_1; ...; X_{16} and $t_{1.1}$;…;$t_{2.5}$

TEST. One point beyond control limits.
Test Failed at points:

13; 20; 30; 40; 50; 60; 80; 100; 114; 118; 123; 128;
133; 134; 138; 140; 143; 147; 151; 156; 161; 166; 167;
171; 173; 176; 180; 184; 189; 194; 199; 200; 207; 221;
227; 233; 247; 256; 263; 264; 267; 271; 275; 279; 283;
287; 394; 424; 428; 432; 435; 443; 448; 451; 454; 457;
458; 460; 463; 464; 466; 467; 469; 471; 472; 474; 475;
477; 478; 480; 481; 482; 484; 485; 487; 488; 489; 493;
495; 515; 517; 521; 525; 529; 636; 647; 657; 658; 663;
664; 665; 669; 673; 679; 683; 687; 691; 695; 699; 703;
707; 711; 714; 715; 718; 719; 721; 723; 724; 727; 730

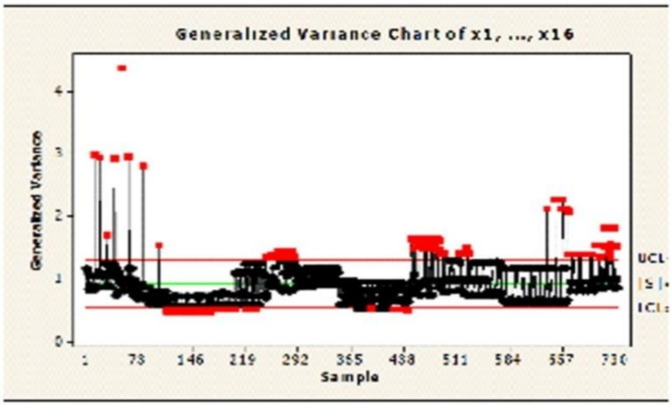

Fig. (1) Generalized variance chart of $X_1;...;X_{16}$and $t_{1.1};\ldots;t_{2.5}$

We can summarize the generalized variance chart of $X_1;\ldots;X_{16}$***and*** $t_{1.1};\ldots;t_{2.5}$***as follows:***

- The lower and upper control limits are 0.528 and 1.312, respectively. Therefore, we expect the generalized variance statistics to fall between 0.528 and 1.312. The center line, or median, is 0.92.
- Test results indicate 111 point through beyond the control limits.
- Test results indicate that the process is in- control for 621 points and out -of control for 111 points. Then the out-of-control rate 15.16% and the in-control rate 84.84%.

(b)Generalized Variance Chart of $y_1;\ldots;y_7$ and $t_{3.1};\ldots;t_{4.5}$

Test Results for Generalized Variance Chart of y_1; ...; $y_7$and $t_{3.1}$; …; $t_{4.5}$

TEST. One point beyond control limits.
Test Failed at points:

91; 114; 130; 150; 200; 247; 250; 256; 258; 263; 264;
267; 268; 271; 274; 275; 278; 279; 282; 283; 286; 287;
551; 703; 714; 715; 718; 721; 724; 727; 730

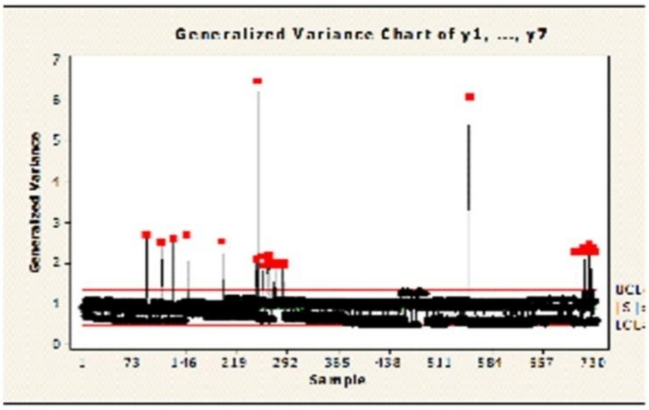

Fig. (2) Generalized variance chart of y_1; ...; $y_7$and $t_{3.1}$; …; $t_{4.5}$

We can summarize the generalized variance chart of* y_1; ...;y_7*and*$t_{3.1}$; …; $t_{4.5}$*as follows:

- The lower and upper control limits are 0.456 and 1.330, respectively. Therefore, we expect the generalized variance statistics to fall between 0.456 and 1.330. The center line, or median, is 0.893.
- Test results indicate 31 point through beyond the control limits.
- Test results indicate that the process is in- control for 701 points and out -of control for 31 points. Then the out-of-control rate 4.23% and the in-control rate 95.77%.

(c) Generalized Variance Chart of Z_1; …;Z_4 and $t_{6.1}$; …;$t_{6.8}$

Test Results for Generalized Variance Chart of Z_1; …;Z_4 and $t_{6.1}$; …;$t_{6.8}$

TEST. One point beyond control limits.
Test Failed at points:

245; 252; 258; 261; 265; 269; 274; 278; 282; 286; 489;
491; 493; 495; 497; 499; 501; 503; 505; 507; 509; 511;
513; 515; 517; 519

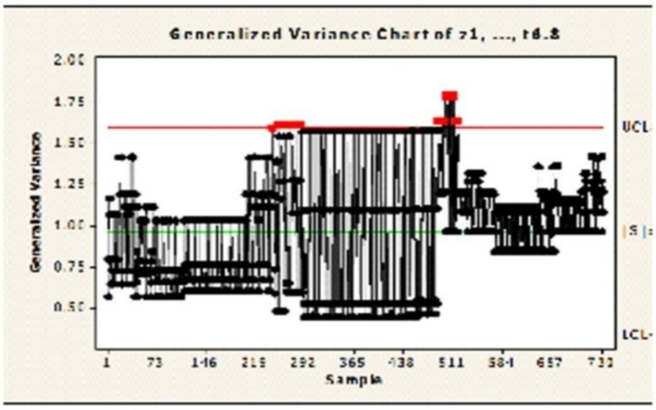

Fig. (3) Generalized variance chart of Z_1; …; Z_4 and $t_{6.1}$; …; $t_{6.8}$

We can summarize the generalized variance chart of Z_1; …; Z_4 and $t_{6.1}$; …; $t_{6.8}$ as follows:

- The lower and upper control limits are 0.342 and 1.594, respectively. Therefore, we expect the generalized variance statistics to fall between 0.342 and 1.594. The center line, or median, is 0.968.
- Test results indicate 26 point through beyond the control limits.
- Test results indicate that the process is in- control for 706 points and out -of control for 26 points. Then the out-of-control rate 3.55% and the in-control rate 96.45%.

5. Results and Conclusions

Generalized variance chart used to determine whether or not the joint process variability (the joint variability that accounts for the variability of each charted variable) for two or variables is in-control.

Generalized variance charts allow us to simultaneously monitor whether the joint variability of two or more related variables is in-control.

5.1 Test Results of the Application:

The application is shown that in generalized variance chart, in the High process stage, test results indicate that the out-of-control percentage 15.16%and the in-control percentage 84.84%, while in the Low process stage, the out-of-control percentage 95.77% and in the Evaporation and Prilling stage, the out-of-control percentage 3.55% and the in-control percentage 96.45%.These results allow us to determine whether the joint process variability is in-control or out-of-control. It is shown that the out-of-control and in-control percentage changes by using difference values of *r*. It was shown that there is a relationship between the value of *r* and the out-of-control percentage, the out-of-control percentage increased by increasing the value of *r*.

5.2 Finally:

The company should use generalized variance chart to determine whether or not the joint process variability for two or more variables is in control.

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